Suppose $X_1, \ldots, X_n$ are iid with the common pdf

$$f(x; \theta) = \begin{cases} \theta x^{\theta-1} & 0 < x < 1 \\ 0 & \text{elsewhere,} \end{cases}$$

(1)

where $\theta > 1$. Consider the hypotheses

$$H_0 : \theta \leq \theta_0 \text{ versus } H_A : \theta > \theta_0 .$$

(2)

1. Determine the UMP test for the hypotheses (2).

2. Using a transformation, show that the test can be conducted using the $\chi^2$ distribution.

3. Carry out your $\chi^2$-test of the hypotheses (2) when $\theta_0 = 2$, for the data:

$$0.590 \ 0.553 \ 0.653 \ 0.599 \ 0.542 \ 0.881 \ 0.827 \ 0.598 \ 0.722 \ 0.625 \ 0.261 \ 0.892 \ 0.199 \ 0.723 \ 0.738$$

Use $\alpha = 0.05$ but conclude in terms of a $p$-value.