Stat 2630: Solution to Homework 12

1. Suppose that $X_1, \ldots, X_{20}$ is a random sample from Poisson with mean 8, and $Y_1, \ldots, Y_{20}$ is a random sample from Poisson with mean 10.

   (a) Simulate the probability that the size .05 two-tailed pooled t-test rejects $H_0 : \mu_1 = \mu_2$.

   (b) Simulate the probability that the size .05 two-tailed Welch t-test rejects $H_0 : \mu_1 = \mu_2$.

   Solution:

```
> n1<-20                      # Sample size for x
> n2<-20                      # Sample size for y                  # Population sd for y
> mu1<-8                    # Population mean for x
> mu2<-10                   # Population mean for y
> nsim<-10000                 # Number of trials
> pval1<-numeric(nsim)        # Storage for p-value of pooled-t
> pval2<-numeric(nsim)        # Storage for p-value of Welch-t
> for(i in 1:nsim){
+   xsim<-rpois(n1,mu1)  # Generate x-data
+   ysim<-rpois(n2,mu2)  # Generate y-data
+   pval1[i]<-t.test(xsim,ysim,alternative="two.sided",var.equal=TRUE)$p.value
+   pval2[i]<-t.test(xsim,ysim,alternative="two.sided",var.equal=FALSE)$p.value
+ }
> cbind(mean(pval1<.05),mean(pval2<.05))
       [,1]   [,2]
[1,] 0.5421 0.5403
```

2. Suppose that $X_1, \ldots, X_{20}$ is a random sample from Poisson with mean 8, and $Y_1, \ldots, Y_{20}$ is a random sample from Poisson with mean θ.

   (a) Plot the simulated probability that the size .05 two-tailed pooled t-test rejects $H_0 : \mu_1 = \mu_2$, using values of θ increasingly farther from 8.

   (b) Overlay the simulated probability that the size .05 two-tailed Welch t-test rejects $H_0 : \mu_1 = \mu_2$

   Solution:

```
 > # Write a function
> welch_sim<-function(nsim=10000, n1=30, n2=30, mu1=8, mu2=8){
+   pval1<-numeric(nsim)        # Storage for p-value of pooled-t
+   pval2<-numeric(nsim)        # Storage for p-value of Welch-t
+   for(i in 1:nsim){
+     xsim<-rpois(n1,mu1)  # Generate x-data
+     ysim<-rpois(n2,mu2)  # Generate y-data
+     pval1[i]<-t.test(xsim,ysim,alternative="two.sided",var.equal=TRUE)$p.value
+     pval2[i]<-t.test(xsim,ysim,alternative="two.sided",var.equal=FALSE)$p.value
```

```
+   }                              # End of for() loop
+   return(c(mean(pval1<.05),mean(pval2<.05)))
+ }                              # End of function()
> # Call the function
> welch_sim(10000,20,20,8,4)
[1] 0.9998 0.9998
> welch_sim(10000,20,20,8,5)
[1] 0.9552 0.9549
> welch_sim(10000,20,20,8,6)
[1] 0.6435 0.6418
> welch_sim(10000,20,20,8,7)
[1] 0.2002 0.1993
> welch_sim(10000,20,20,8,8)
[1] 0.0462 0.0458
> welch_sim(10000,20,20,8,9)
[1] 0.1895 0.1888
> welch_sim(10000,20,20,8,10)
[1] 0.5412 0.5397
> welch_sim(10000,20,20,8,11)
[1] 0.8524 0.8512
> welch_sim(10000,20,20,8,12)
[1] 0.9714 0.9714
```

Looks like the power Welch and pooled-t are equal (up to two decimal places)