Stat 4620 Final Exam   **Name:**_____

April 28, 2016

Instructions: Show all solutions.

1. Let $(3.7, 8.6, 11.1, 22.4, 6.2)$ be the oberved values of a random sample of size $n = 5$ from the continuous distribution with density

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

(20 pts.)   (a) Calculate the maximum likelihood estimate of $P[X > 12.0]$

(20 pts.)   (b) Using the asymptotic distribution of MLE, calculate a 95% confidence interval for λ.

2. Let $\overline{X} = 3.1$ be the observed mean of a random sample of size 25 from a Poisson distribution with mean μ. Conduct a size $\alpha = .05$ test of $H_0 : \mu = 2.0$ versus $H_1 : \mu \neq 2.0$ using

(20 pts.)   (a) $-2\ln(\Lambda)$

(20 pts.)   (b) Rao-score test

(20 pts.)   3. Let $\overline{X} = 3.1$ be the observed mean of a random sample of size 25 from a Poisson distribution with mean μ_1. Let $\overline{Y} = 4.0$ be the observed mean of a random sample of size 30 from a Poisson distribution with mean μ_2. Conduct a size $\alpha = .05$ likelihood ratio test of

$$H_0 : \mu_1 = \mu_2 \text{ versus } H_1 : \mu_1 \neq \mu_2$$

using a rejection rule based on the asymptotic null distribution of $-2\ln(\Lambda)$.