Stat 4620 Homework 9
Due: February 14, 2019

1. Suppose that lifetime of a tire (in miles), say X, is normally distributed with mean θ and standard deviation $\sigma = 5000$. Past experience indicates that $\theta = 30000$ miles, but the manufacturer claims that a new production process has increased the mean lifetime. Given n independent observed values $x_1, x_2, \ldots, x_n$, we want to test

$$H_0 : \theta = 30000 \text{ against } H_1 : \theta > 30000$$

using a rejection rule of the form $\bar{x} \geq c$.

   (a) Write down the formula for the benchmark c so that the test has size $\alpha = .01$.

   (b) Calculate the values of n and c so that the test has size $\alpha = .01$ and power $\gamma(35000) = .98$.

2. Let X have a Poisson distribution with mean θ. Consider the hypotheses

$$H_0 : \theta = 1/2 \text{ versus } H_1 : \theta < 1/2$$

Let $X_1, \ldots, X_{12}$ denote a random sample of size 12, and consider the rejection rule

$$\text{Reject } H_0 \text{ if } X_1 + \cdots + X_{12} \leq 2$$

Recall that if $X_1, \ldots, X_n$ are independent Poisson(θ), then $X_1 + \cdots + X_n$ is Poisson($n\theta$). Run the following R code and answer the questions.

```
theta<-seq(.1,.5,.05)
gam<-ppois(2, theta*12)
plot(gam~theta, pch=" ",xlab=expression(theta), ylab=expression(gamma))
lines(gam~theta)
```

   (a) What is the size of the test?

   (b) What is the power of the test at $\theta = 1/4$?