Binomial Distribution

- **Four Assumptions or properties of a Binomial**
  - The sample consists of a **fixed number of observations**, \( n \).
  - Each observation is classified into one of two mutually exclusive and collectively exhaustive categories, called success and failure.
  - The probability of an observation being classified as a success, \( p \), or a failure, \( 1 - p \), is constant over all observations.
  - The outcome (success or failure) of any observation is independent of the outcome of any other observation.

Binomial probability distribution is a discrete distribution determined by **two parameters**:
- sample size, \( n \)
- probability of success, \( p \)

Random variable \( X = \text{number of successes} \) and its **range** is always \( 0 \) to \( n \)

**Probabilities** are the chance of having certain number/s of successes

**Total** of all probabilities equals \( 1.0 \)

"**Expected Value of X**" = \( E(X) = \text{Mean} = np \) 

**Note**: Do not round, report as a number with a decimal portion

**Standard Deviation** = \( \sqrt{np(1 - p)} \)

**TI-83 functions**: located under **2nd | DISTR**
- 0:binompdf used when \( X = \text{equal} \) to a specific number
- A:binomcdf used when \( X = \text{less than or equal to} \) to a specific number \( \leq \)
- function requires **three arguments** in this order \( (n, p, X) \)
- binomcdf function only accumulates probabilities from zero on up
- **complementary formulas** are used when \( P(X) \) contains \( > \) or \( \geq \)

**Examples**:
- Probability that \( X \) is exactly 3 is written as \( P(X = 3) \) use binompdf \( (n, p, 3) \)
- Probability that \( X \) is at most 3 is written as \( P(X \leq 3) \) use binomcdf \( (n, p, 3) \)

**Note**: For the remaining examples binomcdf is used. However, the probability statement needs to be rewritten first, so that it conforms to the probability accumulation method used by the binomcdf function which is \( \leq \)

- probability that \( X \) is less than 3 is written as \( P(X < 3) \) and rewritten as \( P(X \leq 2) \)
- probability that \( X \) is more than 3 is written as \( P(X > 3) \) and rewritten as \( 1 - P(X \leq 3) \)
- probability that \( X \) is at least 3 is written as \( P(X \geq 3) \) and rewritten as \( 1 - P(X < 2) \)
- Probability that \( X \) is between 3 and 9 exclusively is written as \( P(3 < X < 9) \) and rewritten as \( P(4 \leq X < 8) \) and finally as \( P(X \leq 8) - P(X \leq 3) \)
- Probability that \( X \) is not more than 3 or at least 9 is written as \( P(X \leq 3) \text{ or } P(X \geq 9) \) and rewritten as \( P(X \leq 3) + P(X \geq 9) \) and finally as \( P(X \leq 3) + [1 - P(X \leq 8)] \)

**Calculating Binomial Probabilities**

1. Identify \( n \) and \( p \) and what constitutes a success.
2. Write the probability statement in symbols.
3. List the range of possible values for \( X \), the number of successes, and circle the numbers that are included in the probability.
4. If you only circled one number, then use binompdf and enter the values for \( n, p, \) and \( X \) the number circled.
5. If you circled more than one number and if the numbers circled begin at 0, then use binomcdf and enter the values for \( n, p, \) and \( X \) the **highest** number that you circled.
6. If you circled more than one number, but the circled numbers do not begin at 0, then find the probability for the numbers that were **not** circled and subtract the answer from 1. Use 1 - binomcdf and enter the values for \( n, p, \) and \( X \) the **highest** number that is **not** circled.
7. If you circled more than one number, but the circled numbers are in the **middle** of the range of numbers, then **first** find the probability for the numbers from 0 to the highest number circled. **Second** find the probability for the numbers from 0 to the number just below the lowest number circled and **subtract** this second probability from the first probability to get the final probability for the numbers in the middle.
8. If you circled two different groups of numbers, then you need to find the **probability for each group** separately using one of the methods above. Then you add the **two probabilities** to get the final answer.