14.4 A 95% Confidence Interval for Slope

Is there a linear relationship between $X$ and $Y$? It seems obvious that selling price ($Y$) responds to a car’s mileage ($X$), but in science, relationships are often not too obvious and need confirmation by data. For example, does an individual’s systolic blood pressure ($Y$) tend to increase with their cholesterol level ($X$)? Is there a relationship between one’s total number of years of education ($X$) and income ($Y$)? In this section, we will investigate the strength of linear relationships by looking at the slope estimate. Since the slope represents how much $Y$ responds to changes in the $X$-value, we will calculate a 95% confidence interval for the slope, and examine whether it excludes 0. If it does, then we can rule out the likelihood that the slope is 0. Thus, we conclude that there is a significant linear relationship between $X$ and $Y$.

We start by stating the formula for standard error.

The slope estimate $b$ tends to miss the true value $\beta$ by an amount called the standard error of the slope, denoted SE of $b$, and calculated as

$$
\text{SE of } b = \sqrt{\frac{(1 - r^2)S_y^2}{(n - 2)S_x^2}}
$$

(14.2)

The interval estimate is the familiar $b \pm 1.96(\text{SE})$. It is formally calculated as follows.

A 95% confidence interval estimate for the slope of the regression is given by

$$
b \pm 1.96 \sqrt{\frac{(1 - r^2)S_y^2}{(n - 2)S_x^2}}
$$

(14.3)

If this interval excludes 0, then the likelihood of zero slope is ruled out, and we conclude that there is a significant linear relationship between $X$ and $Y$.

Returning to our Saturn car price example, recall that $b = -.05127$. The standard error of this estimate is

$$
\text{SE of } b = \sqrt{\frac{(1 - [.641]^2)(4079)^2}{(12 - 2)(50989)^2}} = .01942
$$

The 95% confidence interval is

$$
[-.05127 - (1.96)(.01942), -.05127 + (1.96)(.01942)] = [-.09, -.01]
$$

Since this interval excludes 0, we conclude a significant relationship between car mileage and selling price.