10.5 Exercises
Note: All computations are done using the formula given in the textbook, not
the TI-84. Values are rounded to 4-decimal places.

Hypothesis Testing

1.
   a. $\bar{x} = 15.6154; SD = 6.1310; n = 13$
   b. $-4.3846$
   c. $SE_{\bar{x}} = \frac{SD}{\sqrt{n}} = \frac{6.1310}{\sqrt{13}} = 1.7004$
   d. $t = -2.2753$
   e. Significantly before 20%
   f. $H_0: \mu \geq 20$ vs. $H_1: \mu < 20$
   g. $t = \frac{15.6154 - 20}{1.7004} = -2.5786$
   h. $t$
   i. $-0.0121$
   j. Since p-value < 0.05 the observed sample value $\bar{x} = 15.6154$ is significantly
different from 20%. The data provide scientific evidence that elderly
males spend less than 20% of this sleep time in REM.

3.
Given: $\bar{x} = 8350; s = 1840; n = 9$
   a. $SE_{\bar{x}} = \frac{1840}{\sqrt{9}} = 613.3$
   b.
      1. $H_0: \mu \geq 9000$ vs. $H_1: \mu < 9000$
      2. $t = -1.0598$
      3. p-value = 0.1601
      4. Do not reject $H_0$; Frosted Corn will not be discontinued.
5.
Given: \( \bar{x} = 80; \ s = 10; \ n = 15 \)

a. \( SE_{\bar{x}} = \frac{10}{\sqrt{15}} = 2.58 \)

b.
1. \( H_0 : \mu \geq 80 \) vs. \( H_1 : \mu < 80 \)
2. \( t = -3.8730 \)
3. p-value = 0.0008
4. Reject \( H_0 \); The new method is significantly different than the old method.

7.

a. two-independent samples
1. \( H_0 : \mu_c \leq \mu_u \) vs. \( H_1 : \mu_c > \mu_u \)
2. \( t = 1.4017 \)
3. p-value = 0.0993
4. Since p-value = 0.0993 is NOT LESS than 0.05, we fail to reject \( H_0 \). The average salaries of CPA accountants are not significantly higher than the uncertified accountants.

b. Under the same set of hypotheses, the p-value (for \( n_c = 60, \ n_u = 40 \)) = 0.000119 \( \rightarrow \) The p-value is smaller when sample sizes were increased.

c. The Z-test p-value (for \( n_c = 6, \ n = 4 \)) = 0.0784. \( \rightarrow \) The p-value is smaller than the t-test p-value. \( \rightarrow \) The t-test p-value is more appropriate since we only have sample information on the standard deviation, and the sample sizes are small.

d. The Z-test p-value (for \( n_c = 60, \ n_u = 40 \)) = 0.000038. \( \rightarrow \) The p-value is smaller than the t-test p-value. \( \rightarrow \) The t-test p-value is still more appropriate since we only have the sample information about the standard deviation. The Z-test p-value would be a valid approximation only by way of the central limit theorem.

9.

a.) Difference: 4 4 -1 6 5 0 4

b.) \( \bar{x}_d = 3.1429; \ s_d = 2.6095 \)

c.)
1. \( H_0 : \mu_d \leq 0 \) vs. \( H_1 : \mu_d > 0 \)
2. \( t = 3.1865 \)
3. p-value = 0.0094
4. Since the p-value < 0.05, we reject \( H_0 \). There is enough evidence to conclude that the new gasoline additive increases gas mileage.

d.) new \( \bar{x}_d = 3.1429; \ s_d = 2.6095; \) new p-value = 1.4685e-7

e.) Independent t-test p-value = 0.1678 \( \rightarrow \) the p-value for the independent samples t-test is larger. \( \rightarrow \) the p-value for the paired t-test is more appropriate, since the data points are paired by nature (since the observations came from the same cars).

f.) Eyeball: The mileage for the gasoline with additive is consistently higher by 4 for each pair of observations than those for the gasoline without additive.
NOTE: The p-value cannot be computed since the SE is zero.
11.
 a.) $\hat{p} = \frac{118}{250} = 0.472$
 b.) $SE_{\hat{p}} = 0.03157$
 c.)
 1. $H_0 : p = 0.5$ vs. $H_1 : p \neq 0.5$
 2. $Z = -0.8854$
 3. p-value = 0.3759
 4. Since the p-value $> 0.05$, we FAIL TO REJECT $H_0$. The proportion of investors who own real state is 0.5.
 d.) new $\hat{p} = 0.472$; new $SE = 0.00998$; new p-value $= 0.00511$

13.
 a.) $\hat{p} = \frac{31}{120} = 0.2583$
 b.) $SE_{\hat{p}} = 0.03996$
 c.)
 1. $H_0 : p = 0.3$ vs. $H_1 : p \neq 0.3$
 2. $Z = -0.996$
 3. p-value = 0.3192
 4. Since the p-value is NOT LESS THAN 0.05, we FAIL TO REJECT $H_0$. The proportion of customers who buy the contract with their oven is 0.3.

15.
 a.)
 1. $H_0 : p \geq 0.3$ vs. $H_1 : p < 0.3$
 2. $Z = 1.6378$
 3. p-value = 0.9493
 4. Since the p-value is NOT LESS THAN 0.05, we FAIL TO REJECT $H_0$. The proportion of correct guesses is at least 0.3. The person’s faith has some justification.
 b.) new p-value $= 0.9897$; YES, the person’s faith has justification.

17.
 a.) Let 1 indicate $< 30$: $\hat{p}_1 = 0.5582$; $SE_{\hat{p}} = 0.01702$
 b.) Let 2 indicate $\geq 30$: $\hat{p}_2 = 0.4150$; $SE_{\hat{p}} = 0.01405$
 c.)
 1. $H_0 : p_1 = p_2$ vs. $H_1 : p_1 \neq p_2$
 2. $Z = 6.43096$
 3. p-value $= 1.274e-10$
 4. Since the p-value is LESS THAN 0.05, we REJECT $H_0$. The percentage of “prime” credit card holders for the two age groups are significantly different.